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Technical Note

Field synergy equation for turbulent heat transfer and its application

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Abstract

A field synergy equation with a set of specified constraints for turbulent heat transfer developed based on the extremum entransy dissipation principle can be used to increase the field synergy between the time-averaged velocity and time-averaged temperature gradient fields over the entire fluid flow domain to optimize the heat transfer in turbulent flow. The solution of the field synergy equation gives the optimal flow field having the best field synergy for a given decrement of the mean kinetic energy, which maximizes the heat transfer. As an example, the field synergy analysis for turbulent heat transfer between parallel plates is presented. The analysis shows that a velocity field with small eddies near the boundary effectively enhances the heat transfer in turbulent flow especially when the eddy height which are perpendicular to the primary flow direction, are about half of the turbulent flow transition layer thickness. With the guide of this optimal velocity field, appropriate internal fins can be attached to the parallel plates to produce a velocity field close to the optimal one, so as to increase the field synergy and optimize the turbulent heat transfer.

Keywords: Turbulent heat transfer; Field synergy principle; Field synergy equation; Entransy dissipation; Optimization

1. Introduction

Enhancement of convective heat transfer processes can both reduce energy consumption and the cost of heat exchangers. A large number of heat transfer enhancement approaches was developed including twisted tape inserts, internally finned tubes and various three dimensional roughness elements [1–3]. However, most techniques were developed empirically or semi-empirically, especially for turbulent heat transfer.

Guo et al. [4] developed the field synergy principle to improve heat transfer performance, which states that the overall heat transfer rate depends not only on the velocity and the temperature gradient field, but also on their synergy. For a given set of constraints, an optimal velocity field exists which improves the synergy between the velocity and temperature gradient fields to maximize the heat transfer rate. This principle has been validated numerically and experimentally [5,6,9]. During the heat transfer process, heat is conserved, but the entransy [7] is not conserved due to its dissipation by the thermal resistance. Meng et al. [6] then deduced the laminar heat transfer field synergy equation based on the extremum entransy dissipation principle, which was originally referred to as the heat transfer potential capacity dissipation extremum principle. The solution of the laminar field synergy equation was that the optimal velocity field, which has significantly better heat transfer for a given pumping power for laminar heat transfer in a circular tube, was multiple longitudinal flow. The optimal velocity field led to the development of the discrete double inclined ribs (DDIR) tubes [6] and the alternating elliptical axis (AEA) tubes [8] to generate the multiple longitudinal vortexes. Experimental and numerical results showed that the DDIR and AEA tubes effectively enhance laminar heat transfer, but they were not as effective for turbulent heat transfer enhancement, which means that the optimal velocity field predicted by the laminar heat transfer field synergy equation is not quite

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Nomenclature

c_p	specific heat capacity, J kg^{-1} K^{-1}
f	friction factor
j	heat transfer factor
l	distance, m
P	pressure, Pa
Pr	Prandtl number
q	heat flux, W m^{-3}
Re	Reynolds number
T	temperature, K
u, v, w	velocity components in x-, y- and z-directions, m s^{-1}
V	volume, m^3
x, y, z	Cartesian coordinates, m
Z_q	entransy dissipation function, W K^{-1} m^{-3}

Greek symbols

ρ	density, kg m^{-3}
μ	dynamic viscosity, kg m^{-1} s^{-1}
λ	thermal conductivity, W m^{-1} K^{-1}
∇	divergence operator
Π	Lagrange function
Φ	decrement function of the mean kinetic energy, W m^{-3}

Subscripts

if	parallel plates with internal fins
eff	effective
s	smooth parallel plates
t	turbulent
w	wall surface

suitable for turbulent heat transfer optimization. Therefore, a field synergy equation for turbulent heat transfer is needed to guide the design of heat transfer equipment with turbulent flow.

This paper first defines the entransy dissipation function for turbulent heat transfer, which represents the irreversibility of the turbulent heat transfer processes. Then the extremum entransy dissipation principle is used with the continuity equation, the energy equation and a zero-equation turbulent model to develop a field synergy equation for turbulent heat transfer using thermodynamics variational theory. The field synergy equation provides a framework for guiding the design of turbulent heat transfer enhancement techniques. Finally, as an example, the field synergy analysis is used to optimize the turbulent heat transfer between parallel plates to verify the applicability of the turbulent heat transfer field synergy equation.

2. Field synergy equation for turbulent heat transfer

Heat transfer is an irreversible process. When heat is transported from higher temperatures to lower temperatures, heat is conserved, but the heat transfer capacity is not conserved due to dissipation by the thermal resistance, which is defined as entransy dissipation by Guo et al. [7]. The entransy dissipation function is

$$Z_q = \lambda \nabla T \cdot \nabla T. \tag{1}$$

For turbulent heat transfer, the entransy dissipation function can be modified as

$$Z_{qt} = \lambda_{\text{eff}} \nabla T \cdot \nabla T. \tag{2}$$

For a given boundary temperature, heat transfer enhancement means maximizing the heat flux and the entransy dissipation. For a given boundary heat flux, heat transfer enhancement means minimizing the temperature gradient, which results in the minimum heat entransy dissipation. Thus, the optimization objective is to find an optimal velocity field, which has the extremum of entransy dissipation for a given decrement of the time-averaged kinetic energy. Meanwhile, the flow is also constrained by the continuity equation and the energy equation. All these constraints can be removed by using the Lagrange multipliers method to construct a function

$$\begin{aligned}\Pi = \int\int\int_V [&\lambda_{\text{eff}} \nabla T \cdot \nabla T + C_\Phi \Phi + A(\nabla \cdot (\lambda_{\text{eff}} \nabla T) \\ &- \rho c_p \vec{U} \cdot \nabla T) + C_1 \nabla \cdot \vec{U}] \mathrm{d}V,\end{aligned} \tag{3}$$

where A, C_1 and C_Φ are Lagrange multipliers. Because of the different types of the constraints, A and C_1 vary with position, while C_Φ is constant for a given decrement of the mean kinetic energy. Various values of C_Φ lead to various decrement rates of the mean kinetic energy. $\lambda_{\text{eff}} = \lambda + \lambda_t = \lambda + \mu_t/Pr_t$ is the effective thermal conductivity. The turbulent Prantdl number, Pr_t, is 0.9. Φ is the decrement function of the mean kinetic energy

$$\begin{aligned}\Phi = (\mu + \mu_t)\Bigg[&2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \\ &+ \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2\Bigg].\end{aligned} \tag{4}$$

The decrement of the mean kinetic energy equals to the viscous dissipation by the viscous forces, plus the work of deformation of the mean motion by the turbulence stresses, which transform the mean kinetic energy to the turbulence-energy. The turbulent viscosity, μ_t, which is a function of the velocity, can be calculated using the Prandtl's mixing-length model, the Prandtl–Kolmogorov assumption or the k–ε model. However, the Prandtl–Kolmogorov assumption and the k–ε model require additional differential equations for the turbulent kinetic energy (k) and the turbulent dissipation rate (ε) to calculate the turbulent viscosity, which severely complicate obtaining the variation of Eq. (3). Thus, a single algebraic function, which has been validated and used for indoor airflow simulation [10], is used to calculate the turbulent viscosity in this paper

$$\mu_t = 0.03874\rho|\vec{U}|l, \tag{5}$$

where, l is the distance to the nearest wall. As with other zero-equation turbulent models, Eq. (5) is not very theoretically sound, but yields some reasonable results for turbulent flows.

The variational of Eq. (3) with respect to C_1 is the continuity equation and with respect to A is the energy equation. The variational of Eq. (3) with respect to the temperature T is

$$-\rho c_p\vec{U}\cdot\nabla A = \nabla\cdot(\lambda_{\text{eff}}\nabla A) - 2\nabla\cdot(\lambda_{\text{eff}}\nabla T). \tag{6}$$

The variational of Eq. (3) with respect to the velocity component u is

$$\begin{aligned}
&\frac{1}{2C_\Phi}\frac{\partial C_1}{\partial x} + \nabla\cdot(\mu_{\text{eff}}\nabla u) + \frac{\rho c_p A}{2C_\Phi}\frac{\partial T}{\partial x} - \frac{\Gamma u}{2C_\Phi Pr_t}(|\nabla T|^2 \\
&\quad + A\nabla\cdot(\nabla T)) - \frac{\Phi\Gamma u}{2\mu_{\text{eff}}} - \frac{A\Gamma}{2C_\Phi Pr_t}\frac{\partial T}{\partial x} \\
&\quad \times\left[\frac{u}{l}\frac{\partial l}{\partial x} - \left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x}\right)|\vec{U}|^{-2}u + \frac{\partial u}{\partial x}\right] \\
&\quad - \frac{A\Gamma}{2C_\Phi Pr_t}\frac{\partial T}{\partial y}\left[\frac{u}{l}\frac{\partial l}{\partial y} - \left(u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial y}\right)|\vec{U}|^{-2}u + \frac{\partial u}{\partial y}\right] \\
&\quad - \frac{A\Gamma}{2C_\Phi Pr_t}\frac{\partial T}{\partial z}\left[\frac{u}{l}\frac{\partial l}{\partial z} - \left(u\frac{\partial u}{\partial z} + v\frac{\partial v}{\partial z} + w\frac{\partial w}{\partial z}\right)|\vec{U}|^{-2}u + \frac{\partial u}{\partial z}\right] \\
&\quad + \left(\frac{\partial\mu_{\text{eff}}}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial\mu_{\text{eff}}}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial\mu_{\text{eff}}}{\partial z}\frac{\partial w}{\partial x}\right) \\
&\quad + \frac{1}{2C_\Phi}\nabla\cdot\left(\frac{A\Gamma u}{Pr_t}\nabla T\right) = 0,
\end{aligned} \tag{7}$$

where $\Gamma = 0.03874\rho l|\vec{U}|^{-1}$. The variationals of Eq. (3) with respect to the velocity components v and w are similar to Eq. (7). There are four unknown variables and four governing equations including Eqs. (6), and (7), the continuity equation and the energy equation, so the unknown variables can be solved for a given set of boundary conditions. Meanwhile, the flow must also satisfy the momentum equation,

$$\rho\vec{U}\cdot\nabla\vec{U} = -\nabla P + \nabla\cdot(\mu_{\text{eff}}\nabla\vec{U}) + \vec{F}. \tag{8}$$

Comparison with Eq. (8), Eq. (7) is a momentum equation with a special additional volume force, which is referred to as the field synergy equation for turbulent heat transfer. For a given set of boundary conditions, the optimal velocity field, which has a larger heat transfer rate than any other field flow, can be obtained by solving this field synergy equation. Furthermore, various values of the parameter, C_Φ, in the field synergy equation will produce different optimal velocity fields for various decrement rates of the mean kinetic energy.

3. Validation of the zero-equation model for turbulent flow

Since tubes are the common heat transfer geometries, the zero-equation model is validated for predicting flows in tubes, which will be compared to the experimental results and the k–ε model predictions. For simplicity, a repeated segment of tube was studied with the diameter of 20 mm and the length of 80 mm. Water flowing in the tube is assumed periodically fully developed with the Reynolds numbers of 20,000. The CFD program, FLUENT 6.0, was used to solve the conservation equations together with the corresponding boundary conditions. The velocity and pressure were linked using the SIMPLEC algorithm with the convection and diffusion terms discretized using the QUICK format. The total number of the nodes was 200,000.

Fig. 1 shows the friction factors for various Reynolds numbers obtained by experiments and numerical predictions. Compared with the experimental results, the standard k–ε model predicts the friction factor accurately. The zero-equation model predictions are about twice of the experimental results, but the variation trends are the same. Thus, the zero-equation can be considered a reasonable model to predict turbulent flow although there will be some differences between the numerical predictions and the experimental data.

4. Application of the field synergy equation for turbulent heat transfer

The field synergy equation developed in Section 2 can be solved for a constant mean kinetic energy decrement with some given boundary conditions to obtain the optimal velocity field to maximize the heat transfer rate. As an example, flow in a parallel plate channel is studied to illustrate the applicability of the field synergy equation. For simplicity, a repeated segment with the height of 20 mm and the length of 2.5 mm was chosen. Water flowing between the parallel plates is assumed periodically fully developed with a Reynolds number of 20,000. The inlet water temperature is 300 K and the wall temperature is 350 K.

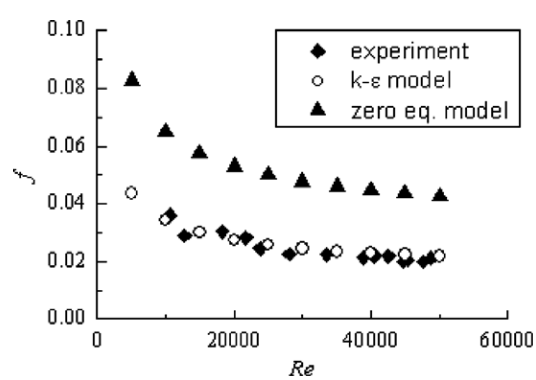

Fig. 1. Friction factor varies with the Reynolds number.

For original results before optimization, the velocity vectors and the temperature gradients are nearly perpendicular to each other, leading to a small scalar product between the velocity vector and the temperature gradient. The field synergy degree [9] is relatively poor. In the computational domain, the heat transfer rate is 1782 W and the decrement of the mean kinetic energy is 4.61×10^{-3} W. The thicknesses of the laminar sublayer and the transition sublayer are 0.116 mm and 0.928 mm, respectively.

The optimized velocity and temperature fields near the upper wall for $C_{\Phi} = -1.5 \times 10^7$ are shown in Figs. 2a and b. There are several small counter-clockwise eddies near the upper wall. The distances between eddies centers is about 0.4 mm and the eddy heights perpendicular to the primary flow direction are about 0.2 mm. There are also several small clockwise eddies near the lower wall for symmetry. For this case, the heat transfer rate is 1887 W and the decrement of the mean kinetic energy is 5.65×10^{-3} W. Compared with the original results before optimization, the heat transfer rate is increased by 6%, while the decrement of the mean kinetic energy is increased by 23%.

For heat transfer in turbulent flow between parallel plates, the temperature gradients in the laminar sublayer are two to three orders of magnitude larger than the gradients far from the wall, which means that the thermal resistance in the laminar sublayer is the dominate resistance in turbulent flow. The conventional heat transfer enhancement viewpoint is to first reduce the dominate resistance to most effectively increase the heat transfer rate. Eddies and disturbances near the wall will increase the velocities, reduce the thermal resistances, and enhance the heat transfer. With the viewpoint of the field synergy principle, the heat transfer field synergy number for the original result was small resulting in a low heat transfer rate. For the optimized results as shown in Figs. 2a and b, in the upper right part of the eddy, the fluid flows towards the wall so the velocity vectors are nearly parallel to the local temperature gradients with the same direction, so the turbulent heat transfer field synergy is improved in this area. In the lower left part of the eddy, the fluid flows away from the wall so the velocity vectors are nearly parallel to the local temperature gradients in the opposite direction, so the turbulent heat transfer field synergy is reduced in this area. However, the magnitude of the temperature gradient in the upper right part is larger than in the lower left part, so the overall field synergy around the entire eddy is improved and the heat transfer is enhanced.

Optimal flow fields for various decrement rates of the mean kinetic energy can be obtained by solving the field synergy equations for various values of C_{Φ}. Fig. 3a–c show

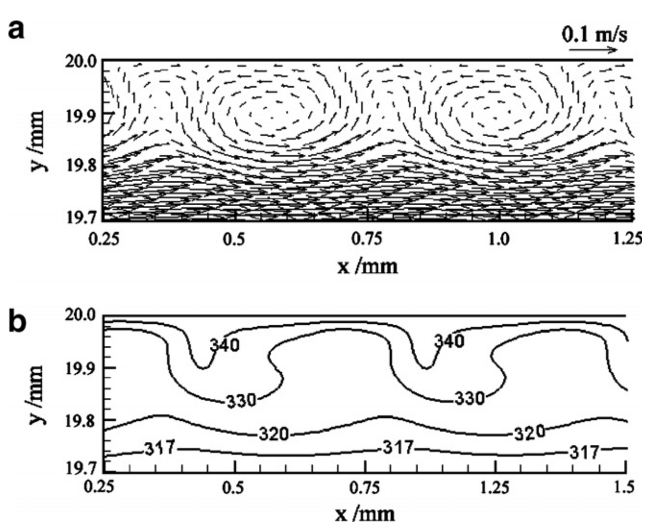

Fig. 2. Optimized results near the wall for $C_{\Phi} = -1.5 \times 10^7$ ($Re = 20{,}000$): (a) velocity vectors; (b) temperature contours.

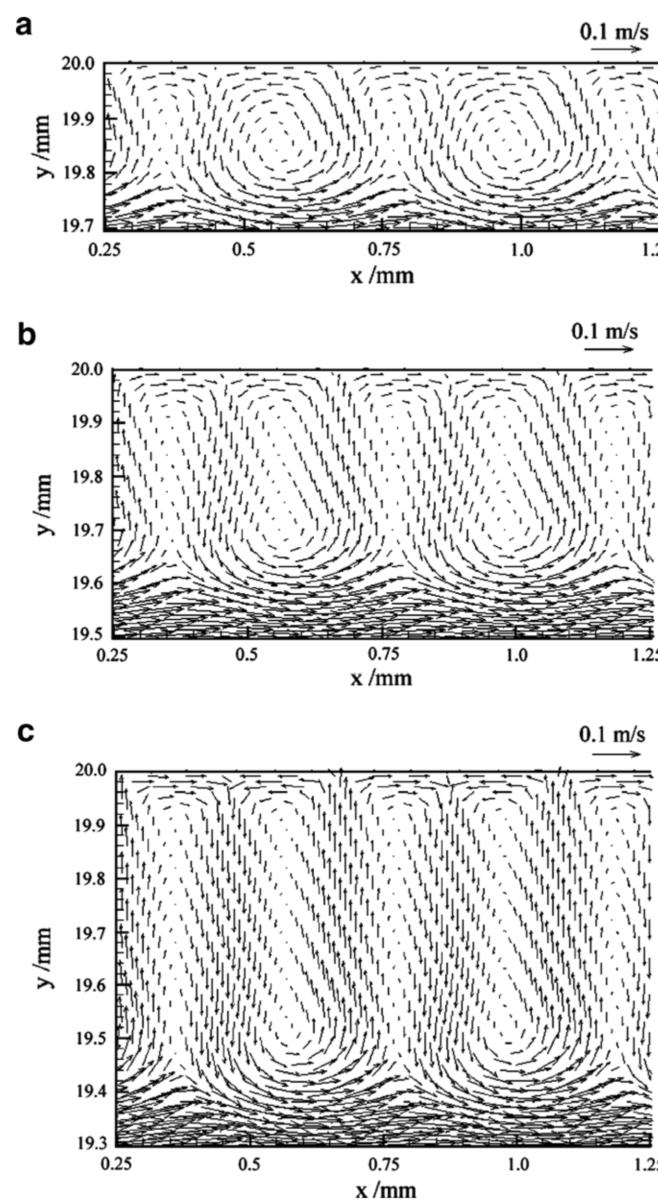

Fig. 3. Velocity distributions near the wall for various C_{Φ} ($Re = 20{,}000$): (a) $C_{\Phi} = -1.3 \times 10^7$; (b) $C_{\Phi} = -1 \times 10^7$; (c) $C_{\Phi} = -5 \times 10^6$.

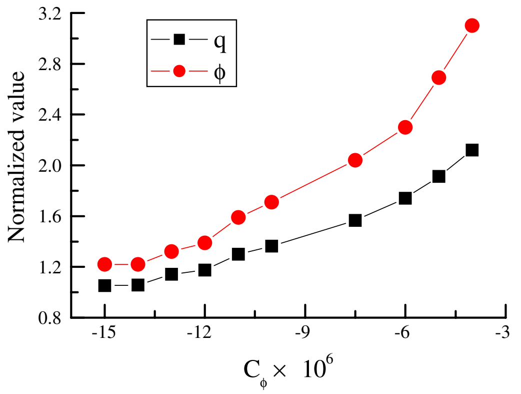

Fig. 4. Relative heat transfer rates and decrement rates of the mean kinetic energy for various C_Φ ($Re = 20{,}000$).

the optimized velocity vectors near the upper wall for $C_\Phi = -1.3 \times 10^7$, -1×10^7 and -0.5×10^6. As C_Φ increases, the rotational speed of the eddy increases and the eddy heights increase from 0.25 mm to 0.4 mm and to 0.6 mm respectively. The eddy heights are all larger than the original laminar sublayer thickness and smaller than the transition layer thickness. Fig. 4 shows the normalized overall heat transfer rates, q, and decrement rates of the mean kinetic energy, Φ, with various C_Φ with both normalized using their respective original values before optimization. As C_Φ increase, both the heat transfer and the decrement rate of the mean kinetic energy increased, but the increases of the decrement of the mean kinetic energy is larger than those of the heat transfer, especially for C_Φ larger than -1×10^7. Thus, for C_Φ of about -1×10^7, the turbulent heat transfer can be effectively enhanced by eddies. For this case, the eddy height is about half the transition layer thickness for the fluid flowing in a smooth parallel plate channel. Thus, small eddies near the wall can effectively enhance turbulent heat transfer especially when the eddy heights are about half the transition layer thickness.

These optimal flow patterns suggest that techniques, such as internal fins, can be introduced to obtain the desired velocity field to enhance the heat transfer and that these fins should have height equal to about half the transition layer thickness as the fluid flow along a smooth wall. To verify these conclusions, the decrement of the mean kinetic energy and overall heat transfer rate, the heat transfer increment rate for a given pumping power (PEC), defined as $(j_{if}/j_s)/(f_{if}/f_s)^{1/3}$, were calculated for several internally finned parallel plate channels whose geometries are summarized in Table 1. Since the turbulent heat transfer will be enhanced most efficiently when the eddy heights are about half of the transition layer thickness, the fin heights in type II were 0.4 mm. Meanwhile, the spaces between fins should be seven to ten times the fin height to generate fully developed eddies [11], so the spaces were 3 mm and the fin widths were 0.6 mm. For comparison, three other internally finned channel structures were also designed as shown in Table 1. The numerical calculations used water at a Reynolds number of 20,000 with an inlet water temperature of 300 K and a wall temperature of 350 K. To more accurately model the actual heat transfer processes, the standard k–ε together, as validated in Section 3, with the enhanced wall functions was used to calculate the turbulent viscosity.

Table 1
Geometries of the internally finned parallel plate channels

Type	Width (mm)	Height (mm)	Spacing between fins (mm)
I	0.6	0.2	1.5
II	0.6	0.4	3
III	0.6	0.8	6
IV	0.6	1.6	12

Table 2
Mean kinetic energy decrements, heat transfer rates and PEC for the internally finned plates for $Re = 20{,}000$

Type	Decrement of the mean kinetic energy (W/m)	Heat transfer (W/m)	PEC
Smooth	0.94	249,900	
I	1.13	333,470	1.25
II	2.29	589,759	1.75
III	4.99	581,750	1.33
IV	10.21	554,884	1.00

The numerical results are listed in Table 2 for the four internally finned parallel plate channels, the decrement of the mean kinetic energy increases with increasing the fin height, while the heat transfer first increases but then decreases for fin heights larger than 0.4 mm, which results in the PEC first increasing and then decreasing. For fins shorter than the transition layer thickness, the internal fins form an eddy near the wall which effectively increases the heat transfer rate especially for fin heights about half the transition layer thickness, which results in the PEC larger than 1. However, when the fins are taller than the transition layer thickness, the internal fins also increase the heat transfer rate, but greatly increase the decrement of the mean kinetic energy, which results in the PEC smaller than 1. These trends are similar to the optimal results calculated by the field synergy equation. Therefore, the solution of the turbulent heat transfer field synergy equation predicts the optimal velocity field, which can guide the heat transfer design.

5. Conclusion

The field synergy equation developed from the extremum entransy dissipation principle together with the continuity and energy equations and a zero-equation turbulence model was solved to predict the optimum turbulent heat transfer flow field. The field synergy equation for turbulent heat transfer is similar to the momentum equation with an additional volume force term. The optimal flow field predicted by the field synergy equation leads to the maximum heat transfer rate in a fixed flow domain for a given decrement of the mean kinetic energy. Variation of the additional volume force term in the field synergy equation leads to different optimal velocity fields for different decrement rates of the mean kinetic energy.

For turbulent flow between parallel plates, the generation of small eddies near the walls effectively enhances the turbulent heat transfer, especially when the eddy heights perpendicular to the primary flow direction are about half the transition layer thickness for a smooth plate. The predicted optimal flow pattern in the parallel plate channel was then used to design appropriate internal fins in the optimal locations to improve the synergy between the velocity and temperature fields to effectively enhance the turbulent heat transfer.

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References

[1] R.L. Webb, Principles of Enhanced Heat Transfer, John Wiley Sons, New York, 1994.

[2] A.E. Bergles, Some perspectives on enhanced heat transfer – second generation heat transfer technology, J. Heat Transfer 110 (1988) 1082–1096.

[3] A.E. Bergles, Heat transfer enhancement – the encouragement and accommodation of high heat fluxes, J. Heat Transfer 119 (1997) 8–19.

[4] Z.Y. Guo, W.Q. Tao, R.K. Shah, The field synergy (coordination) principle and its applications in enhancing single phase convective heat transfer, Int. J. Heat Mass Transfer 48 (2005) 1797–1807.

[5] W.Q. Tao, Y.L. He, Q.W. Wang, Z.G. Qu, F.Q. Song, A unified analysis on enhancing single phase convective heat transfer with field synergy principle, Int. J. Heat Mass Transfer 45 (2002) 4871–4879.

[6] J.A. Meng, X.G. Liang, Z.X. Li, Field synergy optimization and enhanced heat transfer by multi-longitudinal vortices flow in tube, Int. J. Heat Mass Transfer 48 (2005) 3331–3337.

[7] Z.Y. Guo, H.Y. Zhu, X.G. Liang, Entransy – a physical quantity describing heat transfer ability, Int. J. Heat Mass Transfer 50 (2007) 2545–2556.

[8] J.A. Meng, X.G. Liang, Z.J. Chen, Z.X. Li, Experimental study on convective heat transfer in alternating elliptical axis tubes, Exp. Therm. Fluid Sci. 29 (2005) 457–465.

[9] J.A. Meng, Z.J. Chen, Z.X. Li, Z.Y. Guo, Field-coordination analysis and numerical study on turbulent convective heat transfer enhancement, J. Enhan. Heat Transfer 12 (2005) 73–83.

[10] Q. Chen, W. Xu, A zero-equation turbulence model for indoor air flow simulation, Energy Build. 28 (1998) 137–177.

[11] C.F. Ma, W.Z. Gu, Y.M. Zhang, J.R. Shen, Heat Transfer Enhancement, Science Press, Beijing, 1990.